FEFCO RECOMMENDATION N°. 107
(August 2006)

Calculation of the Critical Difference between measurements from Repeatability (s_r) and from reproducibility (s_R) along ISO 5725

This FEFCO recommendation is one of a series numbered from 101 upwards, which gives guidance to FEFCO Members appropriate to the issue as described in the title, in practical matters dealing with production, or customer-related problems. It is hoped that it will provide a uniform means of operation, for example in a comparative study of a problem.
The issuing body is the FEFCO Standards Committee working under the auspices of the FEFCO Board.
The FEFCO Recommendations are supplementary to the internationally recognized FEFCO Testing Methods. The latter will continue to be developed for testing corrugated boards products.

Revised version of original dated September 2005

BACKGROUND

1 – Two series of measurements in one laboratory

Let:

n_1 measurements giving a mean of $\bar{y}_1$

n_2 measurements giving a mean of $\bar{y}_2$

The estimate of the standard deviation of the difference $(\bar{y}_1 - \bar{y}_2)$ is

$$s = \sqrt{s_r^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

where s_r is the repeatability.

The critical difference for $|\bar{y}_1 - \bar{y}_2|$ at 95 % confidence is :

$$C.D = 1{,}96\ s = 1{,}96\sqrt{s_r^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 1{,}96 \times \sqrt{2}\sqrt{s_r^2\left(\frac{1}{2n_1} + \frac{1}{2n_2}\right)}$$

$$C.D = 2{,}77\sqrt{s_r^2\left(\frac{1}{2n_1} + \frac{1}{2n_2}\right)}$$

2 – Two series of measurements in two laboratories

Let :

n_1 measurements in laboratory 1 giving a mean of $\bar{y}_1$

n_2 measurements in laboratory 2 giving a mean of $\bar{y}_2$

The estimate of the standard deviation of the difference $(\bar{y}_1 - \bar{y}_2)$ is

$$s = \sqrt{s_L^2 + \frac{1}{n_1}s_r^2 + s_L^2 + \frac{1}{n_2}s_r^2}$$

$$= \sqrt{2\,s_L^2 + s_r^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$= \sqrt{2\left(s_L^2 + s_r^2\right) - 2\,s_r^2\left(1 - \frac{1}{2n_1} - \frac{1}{2n_2}\right)}$$

s_L^2 = estimate of variance between laboratories

Knowing that Reproducibility is : $s_R^2 = s_L^2 + s_r^2$

$$s = \sqrt{2\,s_R^2 - 2\,s_r^2\left(1 - \frac{1}{2n_1} - \frac{1}{2n_2}\right)}$$

The critical difference for $\left|\bar{y}_1 - \bar{y}_2\right|$ at 95 % confidence is :

$$C.D = 1{,}96\,s = 1{,}96\sqrt{2\,s_R^2 - 2\,s_r^2\left(1 - \frac{1}{2n_1} - \frac{1}{2n_2}\right)}$$

$$= \sqrt{\left(1{,}96 \times \sqrt{2}\,s_R\right)^2 - \left(1{,}96 \times \sqrt{2}\,s_r\right)^2\left(1 - \frac{1}{2n_1} - \frac{1}{2n_2}\right)}$$

$$C.D = \sqrt{\left(2{,}77\,s_R\right)^2 - \left(2{,}77\,s_r\right)^2\left(1 - \frac{1}{2n_1} - \frac{1}{2n_2}\right)}$$

Examples

BCT

1 – Two series of n_1 and n_2 measurements in one laboratory

$$C.D = 2{,}77.\, s_r \sqrt{\frac{1}{2n_1} + \frac{1}{2n_2}}$$

Knowing $r.s.d = 100\ S_r / level$
then

$$C.D = 2{,}77.\, \frac{r.s.d \times level}{100} \sqrt{\frac{1}{2n_1} + \frac{1}{2n_2}}$$

- $n_1 = n_2 = 1$ (Comparison of measurements)

$$C.D = 2{,}77.\, \frac{r.s.d \times level}{100}$$

ex.
$level$ = 300 daN
$r.s.d$ = 4,4 %
$C.D = 2{,}77 \times 4{,}4 \times 300 / 100 = 37\ daN$

- $n_1 = n_2 = 10$ (Comparison of means)

$$C.D = 2{,}77.\, \frac{r.s.d \times level}{100} \sqrt{\frac{1}{20} + \frac{1}{20}}$$

ex.
$level$ = 300 daN
$r.s.d$ = 4,4 %

$$C.D. = \frac{2{,}77 \times 4{,}4 \times 300}{100 \times \sqrt{10}} = 12\ daN$$

2 – Two series of n_1 and n_2 measurements in two laboratories

$$C.D = \sqrt{(2{,}77\ s_R)^2 - (2{,}77\ s_r)^2\left(1 - \frac{1}{2n_1} - \frac{1}{2n_2}\right)}$$

Knowing $r.s.d = 100\ S/level$

then

$$C.D = \sqrt{\left(2{,}77\ \frac{r.s.d_R \times level}{100}\right)^2 - \left(2{,}77\ \frac{r.s.d_r \times level}{100}\right)^2\left(1 - \frac{1}{2n_1} - \frac{1}{2n_2}\right)}$$

- $n_1 = n_2 = 1$ (Comparison of measurements)

$$C.D = 2{,}77.\frac{r.s.d_R \times level}{100}$$

ex.

$level$ = 300 daN

$r.s.d_R$ = 8,0 %

$C.D = 2{,}77 \times 8{,}0 \times 300/100 = 67\ daN$

- $n_1 = n_2 = 10$ (Comparison of means)

ex.

$level$ = 300 daN

$r.s.d_r$ = 4,4 %

$r.s.d_R$ = 8,0 %

$$C.D = \sqrt{\left(2{,}77\ \frac{8{,}0 \times 300}{100}\right)^2 - \left(2{,}77\ \frac{4{,}4 \times 300}{100}\right)^2\left(1 - \frac{1}{20} - \frac{1}{20}\right)}$$

$C.D = 57\ daN$